To the Editor:

In the article "Recursive Estimation in Constrained Nonlinear Dynamic Systems," Vachhani et al. (2005), proposed recursive nonlinear dynamic data reconciliation (RNDDR) as a novel modification of the extended Kalman filter (EKF). However, RNDDR is just an implementation of moving horizon estimation (MHE) in a horizon of one, which is known to reduce to EKF. The authors did not consider the widely discussed relationship between MHE and EKF in the open literature. The most common MHE formulation in a horizon of m measurement is

$$\min_{\bar{x}_{k-m+1},\ldots,\bar{x}_k} \quad e_{k-m+1}^T P_{k-m+1|k-m}^{-1} e_{k-m+1} + \sum_{j=k-m+1}^{k} v_j^T R^{-1} v_j + \sum_{j=k-m+1}^{k-1} w_j^T Q^{-1} w_j \qquad (1)$$

subject to

$$\begin{aligned} v_j &= y_j - h(x_j) \\ w_j &= x_{j+1} - f(x_j) \\ e_{k-m+1} &= x_{k-m+1} - \hat{x}_{k-m+1|k-m} \\ x_j \in \mathbb{X};\ w_j &\in \mathbb{W};\ v_j \in \mathbb{V} \end{aligned}$$

where x is the state vector, y is the measurement vector, f and h are nonlinear functions, $v \sim N(0, R)$, $w \sim N(0, Q)$, $\mathbb{X}$, $\mathbb{W}$ and $\mathbb{V}$ are closed convex sets and mean $\hat{x}_{k-m+1|k-m}$ with covariance $P_{k-m+1|k-m}$ represents the arrival cost. The state vector may be augmented with unknown parameters if simultaneous state and parameter estimation is desired. This general MHE formulation for any m can be found in the literature from a decade ago (Robertson and Lee, 1995; Robertson et al., 1996), and appeared in recent articles (Rao and Rawlings, 2002; Robertson and Lee 2002).

When the horizon moves forward, the arrival cost may be updated by any nonlinear estimator that can propagate the conditional mean and covariance with reasonable accuracy. MHE in a horizon of one ($m = 1$) with boxed constraints removed reduces to that specific nonlinear estimator used for arrival cost. Robertson and Lee (1995) indicated that for unconstrained linear systems, regardless of horizon size, Jazwinski (1970) showed the equivalence of MHE and Kalman filter. Hence "the most sensible choice" is $m = 1$. For constrained systems they advocated the use of longer horizons. Robertson et al. (1996) discussed in detail the equivalence between MHE ($m = 1$) and nonlinear filters, such as the EKF, iterated EKF and higher-order variants of EKF. The same article also notes that for $m = 1$ and linear h, Eq. 1 is a linear least-squares problem because the nonlinear model f is not present. The advantages of $m > 1$ include (1) reducing the influences of approximate arrival cost, (2) linearization in EKF can be based around smoothed estimates, and (3) robustness to plant model mismatch (Robertson et al., 1996; Rao and Rawlings, 2002).

Vachhani et al. (2005) claimed to derive a novel RNDDR for state and parameter estimation based on only the current measurement. But the RNDDR formulation on page 951 is simply MHE for $m = 1$, which was noted previously as a linear least-squares problem. They concluded that recent literature (citing one technical report) suggested superiority of MHE over EKF because EKF lacks constraints and in RNDDR they mitigated the disparity. They claim "we have demonstrated that incorporation of bounds and constraints is not as severe a problem as perceived in literature, and a minor modification of the EKF formulation can handle bounds and constraints." However, that proposed modification, as a special case of MHE, has been known and argued against a decade ago and also in many recent works, Vachhani et al. did not refer to any of the most important articles on MHE published before 2005, except for a technical report. In a subsequent article (Vachhani et al., 2006) the results of RNDDR on a batch reactor simulation exhibit poor convergence. For the same example, Haseltine and Rawlings (2005) reported better results with MHE in longer horizons making a case for not using MHE with $m = 1$.

The main theme presented in Vachchani et al., (2005) is that RNDDR is recursive. It is doubtful because MHE is not a recursive estimator. In MHE the estimates obtained at one sampling time $\bar{x}_{k-m+1}, \ldots, \bar{x}_k$ are not needed for estimating $\bar{x}_{k-m+2}, \ldots, \bar{x}_{k+1}$. The arrival cost is updated separately by an approximate recursive filter such as the EKF. Note that in principle, MHE is not responsible to provide arrival cost information to itself, that is supplied from outside as a summary of historical information. However, if MHE results are used to update arrival cost, the technique becomes iterative rather than recursive. Specifically for MHE in a horizon of one, at time k, past EKF estimate $\hat{x}_{k-1|k-1}$ propagates to $\hat{x}_{k|k-1}$, which is used for MHE estimate $\bar{x}_{k|k}$. Then EKF updates $\hat{x}_{k|k-1}$ to $\hat{x}_{k|k}$. At time $k + 1$, past EKF estimate $\hat{x}_{k|k}$ propagates to $\hat{x}_{k+1|k}$, which is used for MHE estimate $\bar{x}_{k+1|k+1}$. Since MHE estimate $\bar{x}_{k+1|k+1}$ is not a obtained from previous MHE estimate $\bar{x}_{k|k}$ there is no recursion on the estimates. However, in order to improve the accuracy of arrival cost by EKF, one may linearize and propagate using MHE estimates, especially for horizons of more than one, which may seem like recursion but it is actually iteration. This smoothing or iterated update is just one of many ad hoc choices to improve the accuracy of arrival cost. Since RNDDR can be show to be a special case of MHE, it is not novel and a new name for simply choosing a tuning parameter of MHE is not justified. It would be similar to arguing for MHE with $m = 2$ against MHE with $m = 1$.

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